Recitations 11 Previous midterms

Question 1 [Midterm 2017]

Suppose for a given allocation $c = \left\{ \{c_{i,m}\}_{m=1}^{M} \right\}_{i=1}^{I}$ and price vector $p = \{p_m\}_{m=1}^{M}$ that (p, c) is a competitive equilibrium. Suppose you also know that for all $i \in I$ and $\hat{c}_i = \{\hat{c}_{i,m}\}_{m=1}^{M}$ such that $\hat{c}_i \succ_i c_i, \sum_{m=1}^{M} p_m \hat{c}_{i,m} > \sum_{m=1}^{M} p_m c_{i,m} = \sum_{m=1}^{M} p_m e_{i,m}$ and all $i \in I$ and $\hat{c}_i = \{\hat{c}_{i,m}\}_{m=1}^{M}$ such that $\hat{c}_i \succeq_i c_i, \sum_{m=1}^{M} p_m \hat{c}_{i,m} \geq \sum_{m=1}^{M} p_m c_{i,m} = \sum_{m=1}^{M} p_m e_{i,m}$. (That is, you know for all agents that any bundle the agent strictly prefers, he can't afford, and any bundle he weakly prefers, he can just afford.) Show that c is a Pareto efficient allocation.

Question 2 [Midterm 2017]

Consider a pure exchange economy with two agents, $i \in \{1, 2\}$ and two goods $m \in \{1, 2\}$. There exists a total of four units of each good in the economy. Each agent has identical preferences represented by the utility function $u_i(c_{i,1}, c_{i,2}) = \sqrt{c_{i,1}} + \sqrt{c_{i,2}}$ Agent 1 has an unbounded ability to eat any non-negative amount of either good, but agent 2 can eat at most 1 unit of each good.

a) Sketch the utility possibilities frontier for this economy.

b) Set up the "Social Planner's Problem" for this economy for characterizing the set of Pareto efficient allocations.

c) Are all Pareto efficient allocations solutions to the Social Planner's problem? Are all solutions to the Social Planner's problem Pareto efficient?

Question 3 [Midterm 2017]

Consider a pure exchange economy with two agents, $i \in \{1, 2\}$ and two goods $m \in \{1, 2\}$. Each agent has identical preferences represented by the utility function $u_i(c_{i,1}, c_{i,2}) = (c_{i,1})^2 + (c_{i,2})^2$. Each has an unbounded ability to eat any non-negative amount of either good.

a) Suppose $e_{i,1} = e_{i,2} = 1$ for $i \in \{1, 2\}$. Carefully define an allocation and a competitive equilibrium.

b) Find all competitive equilibria of this economy from these endowments. Are they Pareto efficient?

c) Now suppose agent 1 is endowed with 2 units of good 1 and 1 unit of good 2 , while agent 2 is endowed with 0 units of good 1 and 1 units of good 2. (or $e_1 = (2, 1)$ and $e_2 = (0, 1)$) . Is each agent eating his own endowment a Pareto efficient allocation? Find all competitive equilibria of this economy from these endowments.

Question 1 [Midterm 2018]

Consider a pure exchange economy with two agents, $i \in \{1, 2\}$ and two goods $m \in \{1, 2\}$. There exists a total of 1 unit of each good. Agent 1's preference is represented by utility function $u_1(c_{1,1}, c_{1,2}) = \sqrt{c_{1,1}} + \sqrt{c_{1,2}}$. Agent 2's preference is represented by the utility function $u(c_{2,1}, c_{2,2}) = 0$. Both agents have an unbounded ability to eat any non-negative amount of either good.

a) Sketch the utility possibilities frontier for this economy.

b) Set up the two "Arrow Problems" for this economy,

c) Are all Pareto efficient allocations a solution to each Arrow problem?

d) Are all solutions to each Arrow problem Pareto efficient? If so, prove why so. If not, argue why not.

e) Set up the class of "Negishi Problems" for this economy.

f) Are all Pareto efficient allocations a solution to a Negishi problem? (If so, which ones.)

g) Are all solutions to a Negishi problem Pareto efficient?

Question 2 [Midterm 2018]

Consider a pure exchange economy with two agents, $i \in \{1,2\}$ and two goods $m \in \{1,2\}$. There exists a total of 4 units of each good. Both agents' preference are represented by utility function $u_i(c_{i,1}, c_{i,2}) = \sqrt{c_{i,1}} + \sqrt{c_{i,2}}$. Agent 1 has an unbounded ability to eat any non-negative amount of either good. Agent 2, on the other hand, cannot eat more than 3 units of either good.

a) Suppose $e_{1,1} = 4, e_{1,2} = 0, e_{2,1} = 0$ and $e_{2,2} = 4$. Find the competitive equilibrium from this endowment specification. Is it Pareto efficient? If so, prove. If not, argue why not, and specifically, what part of the standard proof breaks down.

b) Suppose $e_{1,1} = 0, e_{1,2} = 0, e_{2,1} = 4$ and $e_{2,2} = 4$. Find the competitive equilibrium from this endowment specification. Is it Pareto efficient? If so, prove. If not, argue why not, and specifically, what part of the standard proof breaks down.

Question 3 [Midterm 2018]

Consider a pure exchange economy with two agents, $i \in \{1,2\}$ and two goods $m \in \{1,2\}$. There exists a total of 1 unit of each good. Both agents' preferences are represented by utility function $u_i(c_{i,1}, c_{i,2}) = (c_{i,1})^2 + (c_{i,2})^2$. Both agents have an unbounded ability to eat any non-negative amount of either good.

a) Find the set of Pareto efficient allocations which can be supported as a competitive equilibrium.

b) Find a Pareto efficient allocation which cannot be supported as a competitive equilibrium. Why not? What step of the proof of the 2 nd Welfare Theorem breaks down?

Question 1 [Midterm 2019]

Consider a pure exchange economy with two agents, $i \in \{1,2\}$ and two goods $m \in \{1,2\}$. There exists a total of 3 units of each good in the economy. Agent 1 has preferences represented by the utility function $u_1(c_{1,1}, c_{1,2}) = \min\{2, c_{1,1}\} + \min\{2, c_{1,2}\}$. Agent 2 has preferences represented by the utility function $u_2(c_{2,1}, c_{2,2}) = c_{2,1} + c_{2,2}$. Both agents have an unbounded ability to eat any non-negative amount of either good.

a) Carefully characterize the set of Pareto Efficient allocations for this economy and sketch the utility possibilities frontier for this economy.

b) Set up the ”Social Planner's” (or Negishi) Problem for this economy.

c) Are all Pareto Efficient allocations solutions to the Social Planner's problem? Are all solutions to the Social Planner's problem Pareto Efficient? Explain.

d) Define a Competitive Equilibrium and give the set of Competitive Equilibria for this economy for all possible endowment specifications subject to the aggregate endowment being 3 for each good. Are they all Pareto Efficient? Are any Pareto Efficient. If not, why not?

e) Can all Pareto Efficient Allocations in this environment be supported as a Com- petitive Equilibrium for some set endowments (again where the aggregate en- dowment is 3 for each good)? If not, what assumption of the 2nd Welfare Theorem is violated?

Question 2 [Midterm 2019]

Consider an I agent, 2 good world, with prices are normalized such that $p_1 + p_2 = 1$ with $p_1 \in [0, 1]$. Debreu's existence proof showed, under certain conditions, that a particular correspondence, $P(p)$, mapping $p \in [0, 1]$ to subsets of [0,1] was guaranteed to have a fixed point p^* such that $p^* \in P(p^*)$ and that $z_m(p^*) = 0$ for all goods m where $z_m(p^*)$ is the excess demand for good m

a) Suppose $p_1 > 0$ and $p_2 = 0$. What is $P(p)$?

b) Suppose $p_2 > 0$ and $p_1 = 0$. What is $P(p)$?

c) Suppose $p_m > 0$ for all m and that $z_1(p) > z_2(p)$. What is $P(p)$?

d) Suppose $p_m > 0$ for all m and that $z_1(p) < z_2(p)$. What is $P(p)$?

e) Suppose $p_m > 0$ for all m and that $z_1(p) = z_2(p)$. What is $P(p)$?